Q. 2 Qn 20.

Find the sum

$$\sum_{n=4}^{\infty} (1/3)^n = (1/3)^4 + (1/3)^5 + \cdots = (1/3)^4 \left(1 + 1/3 + (1/3)^2 + \cdots\right)$$

geometric series.

$$= \frac{(1/3)^4}{1 - 1/3}$$

$$= 1/54.$$

Qn 28

Let h_0 = height ~~after~~ before first bounce = 10 ft

h_1 = height after first bounce = $10(3/4)$ ft.

h_2 = height after second bounce = $10(3/4)^2$ ft.

if h_n is the height of the nth bounce then

(a) $h_n = 10(3/4)^n$.

(b)

Let D_1 be the distance travelled at first bounce

$D_1 = 10$ ft. ($h_0 = 10$ ft.)

has

D_2 ; distance the ball travelled after second bounce.

$$D_2 = h_0 + 2h_1 = 10 + 2 \cdot 10\left(\tfrac{3}{4}\right) = 25 \text{ ft}.$$

Similarly $D_3 = h_0 + 2h_1 + 2h_2$

$$= 10 + 2 \cdot 10\left(\tfrac{3}{4}\right) + 2 \cdot 10\left(\tfrac{3}{4}\right)^2$$

$$= 25 + 2 \cdot 10\left(\tfrac{3}{4}\right)^2 = 36.25 \text{ ft}.$$

and $D_4 = 36.25 + 2h_3$

$$= 36.25 + 2 \cdot 10\left(\tfrac{3}{4}\right)^3$$

$$= 44.69 \text{ ft}.$$

(c) after the n^{th} bounce the total distance travelled by the ball will be:

$$D_n = h_0 + 2h_1 + 2h_2 + \cdots + 2h_{n-1}$$

$$= 10 + 2 \cdot 10\left(\tfrac{3}{4}\right) + 2 \cdot 10\left(\tfrac{3}{4}\right)^2 + \cdots + 2(10)\left(\tfrac{3}{4}\right)^{n-1}$$

$$= 10 + 2 \cdot 10\left(\tfrac{3}{4}\right)\left[1 + \tfrac{3}{4} + \left(\tfrac{3}{4}\right)^2 + \cdots + \left(\tfrac{3}{4}\right)^{n-2}\right]$$

geometric series

$$= 10 + 15\left(\frac{1 - \left(\tfrac{3}{4}\right)^{n-1}}{1 - \tfrac{3}{4}}\right)$$

$$= 10 + 15\,\frac{\left(1 - \left(\tfrac{3}{4}\right)^{n-1}\right)}{\tfrac{1}{4}} = 10 + 60\left(1 - \left(\tfrac{3}{4}\right)^{n-1}\right)$$